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DFL AS A BIASED ESTIMATOR OF FINANCIAL RISK

Introduction

The degree of financial leverage (DFL) is defined as the relative change in earnings after taxes (EAT), caused by a 1% change in operating profit (earning before interest and taxes, EBIT):

$$DFL = \frac{\Delta\% EAT}{\Delta\% EBIT} = \frac{\dfrac{EAT - EAT_B}{EAT_B}}{\dfrac{EBIT - EBIT_B}{EBIT_B}} = \frac{EBIT_B}{EBIT_B - Int} = \frac{EBIT_B}{EBT_B} \tag{1}$$

where *Int* denotes fixed financial costs and *EBT* is earnings before taxes. A subscript *B* denotes the profit level against which percentage changes are measured.[1] *DFL* is a widely used measure of financial risk and/or financial leverage as it tends to be greater than 1. Financial activity of a company, i.e. taking debt and paying interest against it, magnifies („levers" or „gears up") the *EAT* reaction to a relative change in *EBIT*.

There are, however, numerous problems inherent in *DFL* that make this measure dubious both as a leverage as well as financial risk index. Paradoxically, *DFL* does not have to be even greater than 1. What if a 1% change in *EBIT* results in, say, a mere 0.5% change in *EAT*, the case when $DFL = 0.5$? Earnings volatility does not seem to be levered up by financial activity in such a case at all. To make it worse, *DFL* may also be negative. Any leverage interpretation of $DFL < 1$ is simply awkward, if not impossible.

This is by no means the only problem of *DFL*. As illustrated by (1), *DFL* is a function of $EBIT_B$. This leads to many *DFL* values, one for each $EBIT_B$, for the fixed amount of debt and interest. The existence of many *DFLs* in any given financial risk situation makes the index suspicious as a financial risk measure even if the analysis is limited to cases where $DFL > 1$.

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[1] Note that for (1) to be true, the effective tax rate is assumed to be equal to the marginal tax rate (see H. Dilbeck: *A Proposal for Precise Definitions of „Trading on the Equity" and „Leverage": Comment*. „Journal of Finance", 1962, 17.). For simplicity, we assume no taxes throughout.

Another problem with *DFL* comes from the fact that it is defined by accounting entries. This alone makes the measure of rather limited use to practitioners and investors. Last, but not least, being defined in terms of earnings volatility relative to some predetermined level of profit, *DFL* is a mix of both volatility and reward for this volatility. One can justifiably argue that this is not correct.

The issue of *DFL* deficiencies is not trivial given how much significance in various academic textbooks[2] and professional training materials[3] is still attached to *DFL*. In academic literature *DFL* has gained prominence in research on the trade-off hypothesis between operating and financial leverage initiated by Mandelker and Rhee.[4]

The wide use of the degree of operating leverage, *DOL*, i.e. the ratio of the percentage change in operating profit to the percentage change in sales, and hence "*DFL's* twin" that tends to directly precede *DFL* in many finance books, is another reason for concern. DOL is sometimes claimed to have an impact on the systematic risk.[5] Although *DOL's* origins are clearly rooted in financial analysis and managerial accounting, the measure has gained almost a must status when operating risk is defined in finance books. This partly explains in our opinion why *DFL*, certainly an alien body to the finance field too, has proved so resilient as a measure of financial risk.

The ultimate rationale for the use of *DFL* – one can argue – is delivered by Miller, who used the *DFL* reasoning in his 1990 Nobel Memorial Prize Lecture, later twice reprinted by Journal of Finance and Journal of Applied Corporate Finance under a much telling title „Leverage"[6].

Needless to say, there are also countless examples of *DFL* being treated as a viable risk and/or leverage index in the Polish literature[7].

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[2] See e.g. S. Besley, E.F. Brigham: *CFIN*. South-Western, Centage Learning, 2012; G. Hawawini, C. Viallet: *Finance for Executives, Managing for Value Creation*, Thompson, South-Western Centage Learning, 2011; W.L. Megginson, S.B. Smart, J.R. Graham: *Financial Management*. South-Western Centage Learning, 2010; J.C. Van Horne, J.M. Wachowicz: *Fundamentals of Financial Management*, FT Prentice Hall, Harlow 2005.

[3] See e.g. *Financial Reporting and Analysis, CFA ® Programm Curriculum*, Pearson Learning Solutions, Vol. 2, 2011.

[4] G.N. Mandelker, S.G. Rhee: *The Impact of the Degree of Operating and Financial Leverage on Systematic Risk of Common Stock*. „Journal of Financial and Quantitative Analysis" 1984, 19. To be sure, the enthusiasm towards DFL is not shared by other academic empirical research. We have analyzed 92 articles published from 2000–2011 in most respected finance journals according to Financial Times, in which a term *leverage* is used in either the title, abstract or key words. In no paper (sic!) is DFL used. We have also looked at 30 accounting papers published in top accounting journals – again, no mention about DFL.

[5] See e.g. S. Lumby, C. Jones: *Corporate Finance*, *Theory & Practice*. South-Western Centage Learning, 2011; S.A. Ross, R.W. Westerfield, B.D. Jaffe: *Corporate Finance*. McGraw-Hill, Boston 1999; A. Damodaran: *Corporate Finance, Theory and Practice*. John Wiley & Sons, New York, 1997.

[6] M.H. Miller: *Leverage*, „Journal of Finance" 1991, 46; M.H. Miller: *Leverage*, „Journal of Applied Corporate Finance" 2005, 17.

[7] See e.g. G. Michalski: *Wprowadzenie do zarządzania finansami przedsiębiorstw*. C.H. Beck, Warszawa 2010; W. Sibilski: *Rentowność kapitałów własnych a dźwignia finansowa aspekty teoretyczne oraz przykłady na bazie sprawozdań polskich przedsiębiorstw*, „Zeszyty Naukowe Uniwersytetu Szczecińskiego" 2010, nr 587; *Struktura kapitału przedsiębiorstwa*, ed. D. Zawadzka; *Nowoczesne zarządzanie finan-*

This paper focuses on the problems inherent in the *DFL* formulation in general and on the bias present in *DFL* when the index is viewed as an estimator of the true financial risk measure in particular.

DFL modifications

The problems inherent in *DFL* stem directly from its very definition. Formula (1) shows *DFL* as being a product of four constituent characteristics, i.e. it is: an *elasticity index*, that uses *accounting values* of *wealth change (profit)* reported at some *future point in time* $t = 1$. Unless these features of *DFL* are severely modified, the applicability of the measure remains modest.

It can be shown that the switch from the wealth change, i.e. profit perspective, to the total wealth perspective, can successfully resolve the problem of lower than 1 values of *DFL*. In this new disguise, *DFL*, denoted now as DFL_W, is defined in terms of total enterprise (EV) and equity (E) values rather than their incremental changes in wealth as represented by operating and net profit respectively:

$$DFL_W = \frac{\Delta\% EV}{\Delta\% E} = \frac{\frac{E - E_B}{E_B}}{\frac{EV - EV_B}{EV_B}} = \frac{EV_B}{E_B} = 1 + d_1 \qquad (2)$$

where $d_1 = D_B/E_B$ is debt-to-equity ratio at $t = 1$ and a subscript B continues to denote the base value, this time it is the base value of wealth against which wealth percentage changes are calculated. The question DFL_W answers is: how much equity value is going to change, should the change in the enterprise value be 1%. DFL_W, unlike *DFL*, is bound to be greater than 1 if only $E_B > 0$.

Numerical example

Let us illustrate our findings with the help of a numerical example. Let the initial value of assets be $EV_0 = 100$. The assets are partly financed with debt $D_0 = 50$, so that debt-to-

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sami przedsiębiorstwa, chapter 4, C.H. Beck, Warszawa 2009; B. Pomykalska, P. Pomykalski: *Analiza finansowa przedsiębiorstwa*, Wydawnictwo Naukowe PWN, Warszawa 2007; A. Rutkowski: *Zarządzanie finansami*, PWE, Warszawa 2007; A. Żwirbla: *Dźwignia finansowa – próba krytyki oraz syntezy poglądów (artykuł dyskusyjny)*, „Zeszyty Teoretyczne Rachunkowości" 2007, 41; T. Dudycz: *The Different Faces of Leverage*, SSRN, http://ssrn.com/abstract = 950554, 2006; M. Sierpińska, T. Jachna: *Ocena przedsiębiorstwa według standardów światowych*, Wydawnictwo Naukowe PWN, Warszawa 2004; T. Dudycz: *Pomiar efektywności przedsiębiorstwa w stosunku do zainwestowanego kapitału.* „Rachunkowość" 2001, 4; A. Duliniec: *Struktura i koszt kapitału w przedsiębiorstwie*, Wydawnictwo Naukowe PWN, Warszawa 2001; M. Jerzemowska: *Kształtowanie struktury kapitału w spółkach akcyjnych*, Wydawnictwo Naukowe PWN, Warszawa 1999; L. Bednarski: *Analiza finansowa w przedsiębiorstwie*, PWE, Warszawa 1999; L. *Analiza finansowa w zarządzaniu przedsiębiorstwem, tom 1*, ed. L. Bednarski, T. Waśniewski, Fundacja Rozwoju Rachunkowości w Polsce, Warszawa 1996.

equity at $t = 0$ is $d_0 = D_0/E_0 = 1$. The interest rate charged for debt taken is $i = 10\%$. If the base value of *EBIT* is $EBIT_B = 20$ then, according to (1), profit based *DFL* equals 20/15 = 1.33 (see row C in table 1), while wealth based DFL_W, according to (2), amounts to DFL_W = 120/65 = 1.85 (see row G in table 1).

However, if the base for *EBIT* was, say, 2 or –5, then profit based *DFL* is equal to –0.67 and 0.5 respectively. Any change on the operating level is accompanied by less than proportional reaction of profit on the net level (see rows A and B in table 1). The conclusions change when the wealth based analysis is performed. For $EBIT_B = 2$, $DFL_W = 2.17$, while for $EBIT_B = -5$ $DFL_W = 2.38$ (see row E and F in table 1), i.e. unlike the profit based indexes, the wealth based indexes are indeed greater than 1. The leverage effect that is not diagnosed by *DFL* is successfully identified by DFL_W.

Being phrased in terms of cum profit wealth, DFL_W is simply a more comprehensive index than the index based on profit, i.e. the fraction of wealth only. Indeed, the latter is merely a component of the former (as shown below in 3), which may well explain why *DFL* breaks where DFL_W does not.

$$DFL_w = x_1 \times (1 + d_0) + x_2 \times DFL \qquad (3)$$

where the weights $x_1 = E_0/E_B$ and $x_2 = EAT/E_B$ are determined by the size of initial equity and profit contribution to the $t = 1$ value of the equity base respectively.

Equation (3) can be verified with our numerical example. $DFL_w = 1.85$ is a weighted average of $1 + d_0 = 2.0$ and $DFL = 1.33$, with the weights amounting to $w_1 = 77\%$ and $w_2 = 23\%$ (net profit contributes around 23% = 15/65 to the base equity value at $t = 1$). More importantly, profit based $DFL = -0.67$ for $EBIT_B = 2$ and $DFL = 0.5$ for $EBIT_B = -5$, both lower than 1, translate into DFL_W greater than 1 because they are combined with $1 + d_0 = 2 > 1$, whose weight is bound to be positive.

Table 1

Table 1. Profit and wealth based DFL

	$EBIT_B$	EAT_B	DFL	EBIT	EAT	Δ%EBIT	Δ%EAT	DFL
A	–5.0	–10.0	0.50	32.0	27.0	–740.0%	–370.0%	0.50
B	2.0	–3.0	–0.67	32.0	27.0	1500.0%	–1000.0%	–0.67
C	20.0	15.0	1.33	32.0	27.0	60.0%	80.0%	1.33
D	40.0	35.0	1.14	32.0	27.0	–20.0%	–22.9%	1.14
	EV_B	E_B	DFL_W	EV	E	Δ%EV	Δ%E	DFL_W
E	95.0	40.0	2.38	132.0	77.0	38.9%	92.5%	2.38
F	102.0	47.0	2.17	132.0	77.0	29.4%	63.8%	2.17
G	120.0	65.0	1.85	132.0	77.0	10.0%	18.5%	1.85
H	140.0	85.0	1.65	132.0	77.0	–5.7%	–9.4%	1.65

In my recent paper on *DFL*[8], I show that in its standard unmodified version (see equation 1), rather than being a leverage or financial risk index, *DFL* can be interpreted as a language convention used to communicate various (future) business outcomes. There are many *DFLs* in any given capital structure situation, each representing a different language defined by the choice of $EBIT_B$. All bases are mathematically legitimate, so all languages generated by those bases are mathematically legitimate too in that they unambiguously communicate the same information even if not all languages have simple intuitive interpretation. For example, if one wants to flag the operating profit of 32 that leads to the net profit of 27, one can use the following alternative ways to do it (see table 1):

- a 60% increase in the operating profit from 20 to 32 leads to levered 80% growth in the net profit from 15 to 27 ($EBIT_B$ = 20 and DFL = 1.33);
- a 20% decrease in the operating profit from 40 to 32 leads to a levered 22.9% drop in the net profit from 35 to 27 ($EBIT_B$ = 40 and DFL = 1.14);
- a 1500% increase in the operating profit from 2 to 32 leads to a mere 1000% drop in the net loss from –3 to 27 ($EBIT_B$ = 2 and DFL = –0.67);
- a 740% drop in the operating loss from –5 to 32 leads to a mere 370% drop in the net loss from –10 to 27 ($EBIT_B$ = –5 and DFL = 0.5).

Alternatively, one can express exactly the same information in terms of wealth change volatility using (2) (see table 1 again):

- a 10% increase in the enterprise value from 120 to 132 leads to a levered 18.5% increase in the equity value from 65 to 77 (EV_B = 120 and DFL_W = 1.85);
- a 5.7% decrease in the enterprise value from 140 to 132 leads to a levered 9.4% drop in the equity value from 85 to 77 (EV_B = 140 and DFL_W = 1.65);
- a 29.4% increase in the enterprise value from 102 to 132 leads to a levered 63.8% increase in the equity value from 47 to 77 (EV_B = 102 and DFL_W = 2.17);
- a 38.9% increase in the enterprise value from 95 to 132 leads to a levered 92.5% increase in the equity value from 40 to 77 (EV_B = 95 and DFL_W = 2.38).

Needless to say, despite different values of *DFL* and DFL_W used, the message communicated is the same: the operating profit of 32 is accompanied by the net profit of 25. All the *DFL* problems coming from its multi value nature vanish with this new interpretation – the existence of many equivalent ways to express the same message is perfectly acceptable. Note, in contrast to wealth driven DFL_W, not all profit based languages have a leverage interpretation. Unfortunately, *DFL's* claims to be a legitimate leverage or financial risk measure do vanish with this new interpretation too – it may be possible to have many languages, but it is not possible to have many financial risks in a given unique state of firm's financial activity.

[8] T. Berent: *The Base in the Computation of DFL*, „Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu" 2011, 158.

Ultimately, the reason why *DFL* cannot be regarded in its standard form as a viable risk measure, comes from the fact that the base in *DFL* calculation is chosen arbitrarily. Should one unambiguous candidate for the base be identified, *DFL* claims to be a measure of financial risk would be restored. We argue that the switch from the accounting towards market values do just that. The choice of the base in this new framework seems rather obvious: it is the level of the expected enterprise and equity values that correspond to the expected/required, given the risk taken, rate of return – as determined by any asset equilibrium model, such as *CAPM* or *APT*.

Let us assume that the data input we work with are market rather than book values with the operating profit being now replaced by the change in the enterprise market value, the net profit being substituted by the change in the equity market value and k_U and k_G denote expected rates of the change.

Equation (2) translates then into:

$$DFL_W = \frac{EV_0 \times (1+k_U)}{E_0 \times (1+k_G)} = (1+d_0) \times \frac{(1+k_U)}{(1+k_G)} \tag{4}$$

Note, the switch to market values secures both an unambiguous base for *DFL* calculation as well as greater than 1 value for *DFL*, even within a wealth change framework of equation (1) rather then a total wealth framework as described by (2). In market values, equation (1) translates into:

$$DFL = \frac{EV_0 \times k_U}{E_0 \times k_G} = (1+d_0) \times \frac{k_U}{k_G} \tag{5}$$

In terms of input from our numerical example, $DFL_W = 2 \times 1.3/1.2 = 1.85$, while DFL $= 2 \times 0.2/0.3 = 1.33$.

Having identified one unique value of DFL (1.33) and DFL_W (1.85), we need to address the last question: how these values relate to the financial risk they are supposed to capture. Which of the two, *DFL* or DFL_W, if any, is a proper financial risk/leverage measure? The issue is debated in the next section.

DFL as a biased estimator of beta and standard deviation increase

As shown above, financial activity raises returns volatility as measured by *DFL*. It also raises the variance and beta of returns. However, the increase in the volatility as measured by *DFL* or DFL_W is not the same as the volatility increase measured by variance or beta. Let stdev_G and stdev_U denote the standard deviation of the geared and ungeared rates of return respectively. Similarly, let β_G and β_U denote the beta of the geared and ungeared rates of return respectively. Assuming no taxes and zero bankruptcy risk, the rate of return on the geared equity r_G is a linear function of the rate of return on the ungeared equity r_U:

$$r_G = (1 + d_0) \times r_U - i \times d_0 \tag{6}$$

From (6) one can show that the ratio of standard deviations as well as the ratio of the betas is equal to:

$$\frac{stdev_G}{stdev_U} = \frac{\beta_G}{\beta_U} = (1 + d_0) \tag{7}$$

The increase in both total risk, measured by standard deviation as well as systematic, undiversifiable risk measured by beta, increases proportionally with the amount of debt taken at $t = 0$.

This is precisely what Hamada[9] and Rubinstein[10] show in the seminal work on the link between asset pricing model and beta. They do point to the equity multiplier $(D_0 + E_0)/E_0$ as the right measure of the increased risk born by a levered equity holder after taking debt. As shown by (7), the equity multiplier is relevant to the Capital Market Line as much as it is to the Security Market Line.

In his 1990 Nobel Memorial Prize Lecture, Merton Miller uses *DFL* to explain what he means by leverage. In his numerical example, Miller focuses on the fact that the rate of return on equity falls by a greater extent (33.3% in the example) than that for on invested capital (25%) and goes on to explain that this is the reason „why we use the graphic term leverage (or the equally descriptive term gearing that the British seem to prefer). And this greater variability of prospective rates of return to leveraged shareholders means greater risk, in precisely the sense used by my colleagues here, Harry Markowitz and William Sharpe".[11] Miller's example quotes *DFL* determined by the rates of return: levered r_G and unlevered r_U, rather than by the level of profits or wealth change. This helps him link *DFL* with the work of Markowitz and Sharpe phrased in the rates of return. For the same reason we also switch in this section to *DFL* featuring the rates of returns rather than wealth or wealth changes.[12]

In his numerical example, Miller uses similar data to our example. He subsequently ends up with *DFL* of 1.33. If he translated this into wealth driven index DFL_W (still phrased in rates of return) he would have got 1.85, a much closer estimate of the current date equity multiplier 2.0 „used by his colleagues Harry Markowitz and William Sharpe". Still, $1 + d_1 = 1.85$ is not the same as $1 + d_0 = 2.0$.

It goes without saying that both $DFL_W = (1 + d_1)$ and *DFL* are bound to be smaller than $(1 + d_0)$ as in the world of risk aversion, the expected rate of return for the geared equity k_G is always higher than that for the ungeared equity k_U. It looks like *DFL* and DFL_W are

9 R.S. Hamada: *The Effect of a Firm's Capital Structure on the Systematic Risk of Common Stock*,"Journal of Finance" 1972, 27.

10 M. Rubinstein: *A Mean-Variance Synthesis of Corporate Financial Theory*, "Journal of Finance" 1973, 28.

11 M.H. Miller: *Leverage*, "Journal of Finance" 1991, 46, p. 482.

12 More on various equivalent forms of DFL that use earnings, earnings per share or rates of return see T. Berent: *Stopień Dźwigni Finansowej DFL – dziesięć metod pomiaru*, „Przegląd Organizacji" 2010, 6.

nothing but downward biased estimators of the true financial risk index $(1 + d_0)$. In fact, (4) and (5) can now be treated as formulas that determine the size of the bias. If the bias for DFL_W, defined as the percentage shortage of DFL_W relative to $(1 + d_0)$, is denoted by $b_W = (k_G - k_U)/(1 + k_G)$, while the bias for DFL, analogously defined, is denoted by $b = (k_G - k_U)/k_G$, then the following is true:

$$b_W = x_1 \times 0 + x_2 \times b \tag{8}$$

where $x_1 = 1/(1 + k_G)$, $x_2 = k_G/(1 + k_G)$ and $x_1 + x_2 = 1$. Equation (8) shows that, understandably, the bias for the wealth driven DFL_W is always smaller than that for DFL. Using our numerical input, $b_w = (k_G - k_U)/(1 + k_G) = 7.7\%$, while $b = (k_G - k_U)/k_G = 0.33$ – DFL_W is lower by 7.7%, while DFL is lower by 33% than the true value of $(1 + d_0) = 2$. One can verify that equation (8) holds.

The difference between DFL and DFL_W derived from market values and benchmarked against the expected rates of return on one hand and $(1 + d_0)$ viewed as the ratio of standard deviations on the other, comes from the fact that the former are the ratios of average relative distances between the rates of return and their expected values $(r_i - k)/k$ (see equations 9 and 10), while the latter is the ratio of average absolute distances $(r_i - k)$. With the help of (6), equations (9)–(10) can be presented in the form similar to the ratio of standard deviations (11):

$$DFL = \frac{\dfrac{r_{Gi} - k_G}{k_G}}{\dfrac{r_{Ui} - k_U}{k_U}} = \frac{\sqrt{\dfrac{\sum_i^N \dfrac{(r_{Gi} - k_G)^2}{k_G^2}}{N}}}{\sqrt{\dfrac{\sum_i^N \dfrac{(r_{Ui} - k_U)^2}{k_U^2}}{N}}} \tag{9}$$

$$DFL_W = \frac{\dfrac{(1 + r_{Gi}) - (1 + k_G)}{1 + k_G}}{\dfrac{(1 + r_{Ui}) - (1 + k_U)}{1 + k_U}} = \frac{\sqrt{\dfrac{\sum_i^N \dfrac{(r_{Gi} - k_G)^2}{(1 + k_G)^2}}{N}}}{\sqrt{\dfrac{\sum_i^N \dfrac{(r_{Ui} - k_U)^2}{(1 + k_U)^2}}{N}}} \tag{10}$$

$$\frac{stdev_G}{stdev_U} = \frac{\sqrt{\dfrac{\sum_i^N (r_{Gi} - k_G)^2}{N}}}{\sqrt{\dfrac{\sum_i^N (r_{Ui} - k_U)^2}{N}}} \tag{11}$$

Hence the bias present in both *DFL* and DFL_W is not a coincidence, it is the recognition of the fact that the measures are intrinsically different. The comparison of (9) against (11), as well as (10) against (11), leads directly to equations (5) and (4) respectively.

This also shows that while the ratio of standard deviations for a linear model such as (6) is identical to the sensitivity measure based on absolute changes (expressed in percentage points), *DFL* and DFL_W are elasticity measures based on relative changes (expressed in percentages). It is trivial to show that any elasticity measure *ELA* can be derived from the corresponding sensitivity measure *SEN* and the base multiple *M*:

$$ELA = SEN / M \tag{12}$$

For *DFL*, the base multiplier is equal to $M = k_G/k_U$, while for DFL_W this multiplier equals $M_W = (1 + k_G)/(1 + k_U)$. This again leads directly to the link between $DFL = ELA$ and $(1 + d_0) = SEN$ shown in (5) and $DFL_W = ELA$ and $(1 + d_0) = SEN$ shown in (4).[13]

So far, we have illustrated two points. Firstly, we have shown that from the mathematical/statistical perspective market value based *DFL* and DFL_W are always downward biased estimators of true financial risk measures as proposed by Markowitz and Sharpe. Secondly, we have shown the source of the bias in terms of methodology: any *DFL* is an elasticity measure rather than a sensitivity index. Now, the time has come to present a financial rather than mathematical or methodological perspective to the problem.

Relating the absolute changes to the expected value of wealth (DFL_W) or wealth change (*DFL*) does unnecessarily mix risk, i.e. the returns volatility, as captured by absolute changes in rates of returns, with reward dimensions, i.e. the reward for this volatility as captured by the expected value change at $t = 1$. After all, the bases in *DFL* and DFL_W do include the expected gains in wealth that are nothing, but the rewards for the risk taken in the first period. We consequently claim that the reason why any *DFL* cannot be regarded as an adequate measure of risk is that it is not a pure risk indicator at all. This is precisely why the bias is registered and this is also the explanation why multiplying *DFL* by the ratio of the bases designed to remove the reward from *DFL*, removes the *DFL* bias too.

[13] The sensitivity measure in the analysis of returns is always $(1 + d_0)$, regardless of whether the analysis is performed from the perspective of the total wealth or the wealth change (see also equation 6). Note also that the sensitivity measure is equal to $(1 + d_0)$ only because the analysis based on rates of returns rather than earnings is free from, what I call, the scale problem.

The removal of the bias is equivalent to the assumption that the time between $t = 0$ and $t = 1$ is arbitrarily small. This allows the volatility to be present straight after the decision to borrow, thus making the reward for the risk taken negligible. This means that the weight allocated to $(1 + d_0)$ in (3) is arbitrarily close to 1, while the weight allocated to *DFL* is practically zero. DFL_W turns to be $(1 + d_0)$ as a result.

Another way to illustrate how the risk-reward knot present in DFL_W should be disentangled is to assume, that any possible changes in market values at $t = 1$ that include the reward for the risk are instantaneously incorporated in the market value at $t = 0$. Although the assumption that resembles that of market efficiency proposed by Fama is rather heroic, it helps to analyse the sheer return volatility at $t = 0$ without the need for the assumption that the time period from $t = 0$ to $t = 1$ is virtually zero.

Should the market enterprise value change from the expected level by $X\%$ at $t = 1$, then – if this information is already available at $t = 0$ – the enterprise value at $t = 0$ changes also by $X\%$ (see the denominator of equation 13). The nominal change in *EV* at $t = 0$ is claimed by the firm's equity holder (see the numerator of equation 3), so that the equity valuation change is $(1 + d_0)$ times greater than $X\%$ experienced by the whole company. The ratio of these changes at $t = 0$, $\Delta\%E/\Delta\%EV$, equals $(1 + d_0)$ – the equity multiplier $(D_0 + E_0)/E_0$ at $t = 0$:[14]

$$\left(\frac{\Delta\%E}{\Delta\%EV}\right)_0 = \frac{\dfrac{\dfrac{EV_0 \times (1+k_U) \times (1+X\%)}{1+k_U} - EV_0}{E_0}}{\dfrac{\dfrac{EV_0 \times (1+k_U) \times (1+X\%)}{1+k_U} - EV_0}{EV_0}} = 1 + \frac{D_0}{E_0} \tag{13}$$

Should the change in the geared equity at $t = 1$, incorporated in the valuation already at $t = 0$, be calculated directly by discounting geared equity value at $t = 1$ at the appropriate discount rate k_G^*, then the same result as in (13), i.e. $1 + d_0$, can be arrived at differently:

[14] Although, the ratio defined in (13) seems to have a clear elasticity interpretation, it can also be interpreted as a multiplier or even a sensitivity index. It is a multiplier in that it combines two rates of return into one ratio that can be subsequently used in the elasticity and sensitivity analysis. Alternatively, if the analysis is done from the perspective of wealth with the base levels set at 100%, then (13) is simply a sensitivity index that, as the base multiplier is equal 1, happens to be identical to the elasticity measure. The issue is not trivial and certainly deserves a thorough investigation and a separate treatment. I do believe that the confusion surrounding a leverage concept has a lot to do with double or even triple meaning of many leverage indexes derived from elasticity and/or sensitivity analyses.

$$\left(\frac{\Delta\%E}{\Delta\%EV}\right)_0 = \frac{\dfrac{\dfrac{E_0 \times (1+k_G) \times [1+(1+d_1) \times X\%]}{1+k_G^*} - E_0}{E_0}}{\dfrac{\dfrac{EV_0 \times (1+k_U) \times (1+X\%)}{1+k_U} - EV_0}{EV_0}} = 1 + \frac{D_0}{E_0} \tag{14}$$

Equation (15) shows that $DFL_W = 1 + d_1$ can be presented in a similar fashion as $(1 + d_0)$ in (14) with the only difference that k_G^* in (14) is replaced by k_G in (15). Or put it differently, if k_G^* in (14) is replaced by k_G then $(1 + d_0)$ turns to be $(1 + d_1)$:

$$\left(\frac{\Delta\%E}{\Delta\%EV}\right)_0 = 1 + \frac{D_0}{E_0} \neq \frac{\dfrac{\dfrac{E_0 \times (1+k_G) \times [1+(1+d_1) \times X\%]}{1+k_G} - E_0}{E_0}}{\dfrac{\dfrac{EV_0 \times (1+k_U) \times (1+X\%)}{1+k_U} - EV_0}{EV_0}} = 1 + \frac{D_1}{E_1} \tag{15}$$

However, k_G is clearly a wrong discount rate to use, hence $(1 + d_0)$ is a wrong risk measure.

Solving (14) for k_G^*, we can show that a correct discount rate is a weighted average of geared and ungeared expected rate of return:

$$k_G^* = x_1 \times k_G + x_1 \times k_U \tag{16}$$

where the weights are dependent on $X\%$ – the size of the enterprise value change relative to the expected level: $x_1 = 1/[1 + (1 + d_0) \times X\%]$, $x_2 = [(1 + d_0) \times X\%]/[1 + (1 + d_0) \times X\%]$. Assuming equity remains positive, which implies that $x_1 > 0$, the proper discount rate k_G^* is always greater than k_U.

For positive $X\%$, equation (16) suggests that $k_G^* < k_G$. This is because the equity value at $t = 0$ increases, making the risk of levered equity lower, $k_G^* < k_G$. If $X\% < 0$, the value of equity decreases at $t = 0$, hence inflating the relative share of debt financing. The risk of equity increases as a result and $k_G^* > k_G$. The weight x_2 is now negative, the weigh x_1 is greater than 1 and k_G^* rises above k_G as a result. If the positive outcome materialises, the discount rate in (15) is too high, depressing effectively the increased value of equity and hence decreasing artificially the equity multiplier. If the negative outcome materialises, the discount rate is too small, flattering the equity that in fact decreases more than the formula suggest. This again depresses the equity multiple. To summarise, depending on the $t = 1$ outcome, k_G may be a too low or too high estimator of k_G^*, resulting nevertheless in a too low level of the equity multiple in either case.

Conclusions

The degree of financial leverage *DFL* enjoys unwarranted attention among those writing about and/or teaching finance both in Poland and abroad. The fact that the Nobel Prize winner used it in his Nobel Memorial Prize Lecture may serve as testimony of *DFL's* fatal attraction. Although a detailed and rigorous criticism of *DFL*, given its countless versions and misleading interpretations in the literature, deserves a separate treatment, we have attempted in this paper to illustrate why the index actually breaks as a measure of financial risk, even after major modifications. We have used several platforms to argue against the index:

– from the statistical/mathematical perspective, *DFL* is always a downward biased estimator of a true financial risk measure,
– from the methodological perspective, *DFL* is wrong, because it is a measure of relative changes rather than absolute differences in returns,
– from the finance theory perspective, *DFL* breaks, because it bundles risk with reward dimension in one ratio,
– from the finance practice perspective, *DFL* fails, because it implies a wrong discount rate.

Although numerous modifications were suggested in this paper, including algorithms that adjust for the biases inherent in *DFL*, we strongly recommend not using them at all. Our recommendation is simpler: ditch *DFL* altogether, once and for all.

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Summary

Despite its wide use in finance literature, *DFL* is not an adequate measure of either financial leverage or financial risk. Dependent on operating profitability, it produces multiple values, even if firm's financial activity is fixed. As book value driven measure, its applicability in rather limited. Unfortunately, even radical modifications to the index, including its redefinition to market values, that secures unambiguous reading and leverage interpretation, does not prevent *DFL* from being a downward biased estimator of the true financial risk measure. The paper establishes the source and the size of the bias and provides ways to correct for it. The paper ends with the plea to abandon the index altogether.

DFL JAKO OBCIĄŻONY ESTYMATOR RYZYKA FINANSOWEGO

Streszczenie

Pomimo swojej „popularności" w literaturze przedmiotu, wskaźnik *DFL* nie może być uznawany ani za wiarygodny wskaźnik dźwigniowy ani za odpowiedni miernik ryzyka finansowego. Jako zależny od rentowności operacyjnej, przyjmuje on wiele wartości, w tym mniejsze od jeden i ujemne, dla danego stanu działalności finansowej firmy. Zdefiniowanie wskaźnika w oparciu o wartości księgowe sprawia, że nie jest on użytecznym narzędziem w analizie wartości firmy. Nawet po głębokich modyfikacjach i zredefiniowaniu dla wartości rynkowych, zapewniających jednoznaczność odczytu i dźwigniową interpretację, wskaźnik ten okazuje się być ujemnie obciążonym estymatorem prawidłowego miernika ryzyka finansowego. W artykule wskazano na przyczyny i wielkość obciążenia oraz sposoby jego usunięcia. Artykuł zakończony jest apelem o całkowite zaprzestanie używania *DFL* w analizie ryzyka finansowego.