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Control Processes in the Light of Economic Decisions Logistics on the Capital Market

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Abstract. The article presents an outline of control processes in the light of economic decisions logistics on the capital market. A mathematical analysis given in the article shows the control processes of economic entities and directions of optimal control in open systems of feedback. The control process in the technical analysis approach is illustrated by a numerical example from the capital market. In the final part of the article the author refers to a position given in the bibliography which shows the application of an algorithm of dynamic programming for optimization of a long-term investment portfolio of stock on the capital market.

Keywords: control, optimization, technical analysis, programming dynamic, logistics

Introduction

An efficient and effective course of decision- making processes is ensured by numerical methods commonly used in economic logistic or investment – portfolio processes on the capital market. One of the management tools less frequently used by investors is the process of control mainly in the area of optimization processes of constructing long-term investment portfolios.

Control is the term used in the field of cybernetics i.e. the science of controlling relatively isolated economic systems and it means determining the desired condition of the system, which is subsequently corrected with the use of the regulation of all possible deviations on the used resources. In economic decisions implemented on the capital market the control process is useful for defining risk in the technical analyses on the capital market.

1. Controlling economic processes. Optimization logistics of economic decisions

In essence, economic processes are the processes of decision-making. Shaping economic results requires taking a rational approach in all stages of the economic process both in the operational and the strategic aspect. Moreover, it requires an accurate defining of aims and

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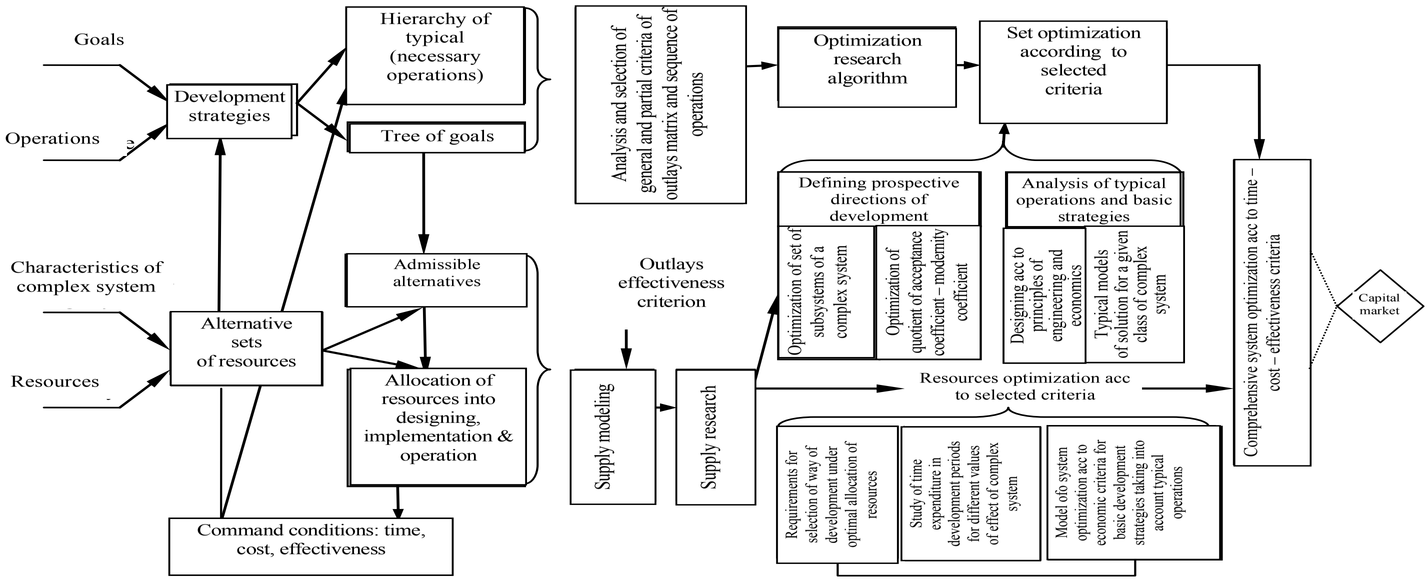


Figure 1. Block diagram algorithm of the optimization of complex economic logistic system planning

Source: study based on the theory of decision – making processes (Sadowski 1960) and the theory of economic processes optimization (Baborski et. al. 1983).

rules of setting criteria for all subsystems of this process logistics according to the general criterion: time- cost – effect. This criterion is directly connected with optimization modelling. Optimal control processes can form the basis for taking decisions on the capital market in the area of strategic investments (Dworecki 1989) (cf. Figure 1).

2. Types of control systems in economic processes

The study entitled *Elements of economic cybernetics* (Baborski et al. 1983: 110–111) some elementary control systems were discussed. Their graphic representation is given in Figures 2 and 3.

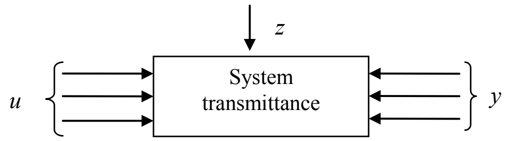


Figure 2. Control object

Source: Baborski et al. (1983): 111

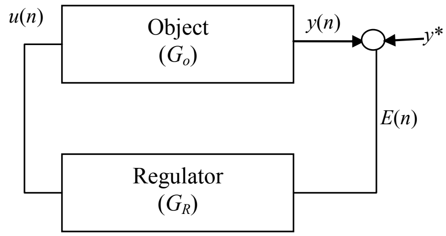


Figure 3. Block diagram of a regulation system

Source: Baborski et al. (1983): 112.

In figures 2 and 3 the following denotations were used,
where:

$E(n)$ – yielded result (income, risk),
G_o – object transmittance
G_R – regulator transmittance,
u – input quantities (control),
y – output quantities for which control state is determined and which depend on control quantities,
y^* – optimal quantities (control),

z – disturbances, whose appearance is independent of the course of the control process and their size and direction (increasing or decreasing the result of control) remains outside the sphere of influence,

ε – regulation error equal to ($y^* - y$).

According to Baborski (1983: 112) control processes can be supported by the model of the transmittance of disturbances in the regulation system of the control system. It is essential to know the course of disturbances of control process in the historic approach (in retrospection of several periods of time) e.g. changes in trends in the technical analysis (boom, bust) (Achelis 1998: 28). Then, knowing the function of the probability distribution $f(z)$, it is possible to define the function generating moments of the random variable (Jędrzejczyk et al. 2004: 19):

– for continuous distribution:

$$M(t) = E(e^{zt}) = \int_{-\infty}^{+\infty} e^{zt} f(z) dz \qquad (1)$$

– for step distribution:

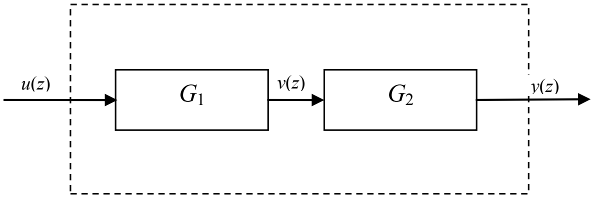
$$M(t) = E(e^{zt}) = \sum_{i=1}^{n} p_i e^{zt} \qquad (2)$$

On the basis of estimated changes in the trends of stock prices formation on the capital market, which were defined in the process of the technical analysis, we get the functions generating the moment of disturbance time in the regulator: $M(t)_0 = e^{0,1t}$ (earlier signals of boom – bust change for $t = 0$) and $M(t)_R = e^{0,25t}$; later realization of the signal of disturbances in the controlled object for $t = 1$). From the observed data it follows that the probability of these changes expressed by estimated functions $M(t)$, amounts to, respectively: for $M(t)_0$: $p_0 = 0{,}85$, for $M(t)_R$: $p_R = 0{,}55$. Hence, the final transmittance z: for $M(t)_0 = 0{,}85e^{0{,}10 \cdot 0} = 0{,}94$ a while for $M(t)_R = 0{,}55e^{0{,}25 \cdot 0} = 0{,}71$.

The value of $G(z)$ is calculated on the basis of the formula (Baborski et al. 1983: 112, Jędrzejczyk et al. 2004: 19):

$$G_{(z)} = \frac{G_{R(z)} \cdot G_{0(z)}}{1 + G_{R(z)} \cdot G_{0(z)}} = \frac{p_R M_R \cdot p_0 M_0}{1 + p_R M_R \cdot p_0 M_0} = \frac{0{,}71 \cdot 0{,}94}{1 + 0{,}71 \cdot 0{,}94} = 0{,}40\,.$$

Formula $G(z)$ describes the transmittance of the system with negative feedback. It can take the form of an equation $E(z) = u(z) - v(z)$, where: $u(z)$ – disturbance input (z) into the system, $v(z)$ – passage from G_1 to G_2 (Fig. 4).



(G_1, G_2 – transmittances of systems).

Figure 4. Series feedback of two systems

Source: Author's own elaboration based on Baborski et al. (1983): 112.

The result $G(z)$ = 0,40 may be interpreted as the probability that the system of object control regulation is at 40 % subject to disturbances whose size cannot be influenced in any way. The investor who takes a decision on portfolio construction under such circumstances of the changing rates of securities (i.e. changes in boom and bust trends) (Carolan 1996; Rothschild 2000) should take into account the fact that the influence of disturbances will be significant and will be equal to $u(z)$ = 0,40 (40%).

Moreover, it should be noted that exiting one system will mean entering another one; the complexity of the new system may vary (Kisperska-Moroń, Krzyżaniak 2009). Such a series of systems' reactions occurs in economic processes logistics. For instance, in the company which follows its strategic development through portfolio optimization on the capital market the effects measured by the increase in capital will be used in earlier periods to purchase new portfolios which include the subsequent capital increment (as a result of the optimal control of capital resources). Regulation systems are considered to be the control of a closed system, while a cybernetic system (with feedback) is an open system.

Coordinates of the control system (u_1, y, z) are variables and each of them characterizes a given quality of the system. In the analysis of economic problems a variety of coordinates can be taken into account however, for practical reasons we need to distinguish the most significant set. For instance when a portfolio system of the capital market is considered and a fundamental analysis of companies is carried out in a complex system one also needs to analyse indebtedness coefficients, efficiency of management coefficients etc. (TMAI concept – Tarczyński 2002). When systems are combined (e.g. a portfolio system on the capital market with a logistic system of a company we need to take into account summing quantities which are of an additive character. However, it is unacceptable to sum non- additive quantities e.g. we cannot sum quantities which constitute a new portfolio system which consists of a set of stock of two earlier constructed portfolios – changeability coefficients of averaging values of probabilities of events occurring (*A Dictionary of Mathematics* 1985) etc. The system behaviour is manifested by changes in the time function and parameters of deviation in time (Murphy 1999). Changes in parameters can be presented in the form of a vector of disturbance. In the theory of systems it is frequently denoted by the Greek letter ε. Such a vector of disturbance of random influence on the economic system is only considered when direct influence on the economic system is either impossible, or is not necessary. For instance, in case of constructing a portfolio of securities correlated positively or negatively it is not necessary to take into account risk for separate sets of stock.

Economic processes which are related to objects such as a manufacturing plant or an enterprise usually take the form of closed control systems (Fig. 4). An open system is presented in Figure 4. When the system such as a capital market is controlled there may appear a tendency to maximize the investor's utility which is described by the square utility function (Tymiński 2011: 272).

Optimal control plays a dominating role in decision-making processes dealing with economic processes which are often reduced to systems of optimal regulations also called extreme systems (Bołtiański 1971; Pońsko 2000). For this kind of decision-making process under conditions for economic entities some other types of regulation systems are also known e.g. a stabilization regulation system. A good example of the application of the stabilization regulation system on the capital market is when the investor purchases a portfolio whose construction has not been changed for a longer period of time. Such a variant of a portfolio is characterized by the high durability of stock. In case of the strategic development of an enterprise a suitable example may be the stable development model (Pluta 2000: 203).

In stabilization, regulation systems the output quantity $y(n)$ takes the form of a logistics constant value (set earlier):

$$y(n) = y^{*}(n) = constans \tag{3}$$

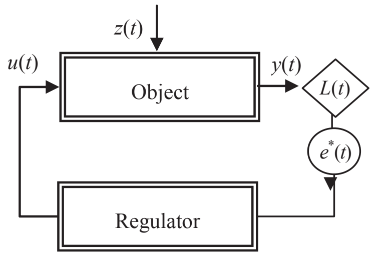
Another example of regulation is the tracking system in which the control quantity is an unknown series of results appearing in $t(n)$ ($n = 1, 2, \ldots$) moments of time. The values which $y^{*}(n)$, will take are independent of the processes taking place inside the logistic system of regulation. They result from the changeability of interaction factors (events) from the outside of the system. In the system of tracking regulation we deal with " inverse" control processes: the regulated (controlled) value "tracks" changes in the control value $u^{*}(n)$. The control cards of quality processes are an example of how the system of tracking regulation is used in management processes.

A somehow simplified example taken from the capital market can be "tracking" diagrams of technical analysis (boom and bust), which provide information for potential regulations of the selection of the investment strategy.

A special variety of the system of tracking regulation is the system of programme regulation. This type of regulation can be exemplified by a strategic portfolio which consists of a set of securities. Such a portfolio is optimal so it enables the company to devise and implement a portfolio logistics strategy in a longer period of time.

3. Processes of optimal control of economic systems

In economic systems a fundamental thing is to take decisions based on optimal control in the system of extreme regulation. In Figure 5 can be seen a block diagram of the system of regulation i.e. optimal control (in a dynamic approach). It can illustrate e.g. maximization of income of the purchased portfolio of securities depending on the risk size. The role of the system can be played by a set of stock in relation to rates of return, depending on the standard deviation assigned to them. (Tymiński 2013). The same example may also have economic sense if it deals with e.g. minimizing expenses on the fuel needed to cover a given distance. Then the subject of regulation will be such a distribution of travelling speed which will allow fuel consumption to stay at the minimum level (or keep to a minimum level of costs). In the portfolio theory this may be a decisive case when the investor, using portfolio diversification, wants to keep a possible steady level of risk.



L – system, object.

Figure 5. Block diagram of the system of extreme regulation in a dynamic approach

Source: Author's own elaboration based on Baborski et al. (1983): 116.

4. Optimal control

Optimal control is defined as the control process which results in the extreme of a given functional on the given area of admissible solutions of the problem. (Bukietyński 1974). In terms of cybernetics such a definitional approach to optimal control (*The Dictionary of Mathematics* 1985) can be reduced to processes which describe the relation between control values u_i (input) and controlled values (output) y_i in the form of a general model (Baborski et al. 1983: 116, 120):

$$Y(n + 1) = g[y(n), u(n)] \quad (4)$$

where:

y – output processes (results of optima control),

g – vector function,

y – n-dimension vector of state and
u – n-dimension control vector of the form:

$$\mathbf{y}(n) = \begin{bmatrix} y_1(n) \\ \vdots \\ y_k(n) \end{bmatrix}, \ \mathbf{u}(n) = \begin{bmatrix} u_1(n) \\ \vdots \\ u_k(n) \end{bmatrix} \tag{5}$$

In model (5) the criterion of the quality of the control process is used while control values (input) are related to only a given area Ω, which limits the possible (admissible) control. Therefore, the optimum (maximum or minimum) value is searched for the following expression:

$$I_N = \sum_{n=0}^{N} g[y(n), u(n)] \tag{6}$$

when

$$u(n) \in \Omega \tag{7}$$

5. Optimal dynamic control

Optimal control is the strategy which consists in finding the extreme value $u^* = \{u(n)\}$, $u(n) \in \Omega$, which is optimum – maximum or minimum. It enables us to achieve the optimum value of a given coefficient of process quality as a function of the model criterion. In order to define an optimal dynamic control series one can use, for example the method of the discrete maximum principle, in case of the maximization of the coefficient value. Thus, a given model is reduced to the form of a structural model with a possibility of using the method of dynamic programming (Baborski et al. 1983: 81, 117) to solve the model. It should be mentioned that functions g represent a suitable control i.e. carry out the regulation system given in the form of a series of discrete values ($u(n)$) – z $y(0)$ to $y(n)$. The strategy which will allow achieving value I_N can be considered the optimal strategy. The same aim can also be achieved when the dynamic programming (Bellman 1957) is applied. Moreover, we can use some other methods of optimal control e.g. the discrimination function (Bołtianski 1971; Gierałtowska 2004: 197).

6. Determining the optimal control signal

The method resulting from the Pontriagin maximum principle (1) is one of the methods of solving the problem of dynamic optimizations (Rumantowski et al. 1984: 72). The problem consists in determining the control vector $u(t)$, which ensures a transition of the object described by equations of state (for subsets included in continuous functions)

$$x_i = f_i(\boldsymbol{x}, \boldsymbol{u}), \quad i = 1, ..., n \tag{8}$$

where is $\mathbf{x}$ – n-dimensional vector of state; $\mathbf{u}$ – m-dimensional vector of admissible control $(\mathbf{u} \in \Omega)$, from the initial state $x_0 = \mathbf{x}(t_0)$ to the final state $x_k = \mathbf{x}(t_k)$, under the minimum of the quality coefficient

$$J = \int_{t_0}^{t_k} f_0(x,u)dr \tag{9}$$

In the above method the temporary loss function f_0 is included into the initial set of equations (8) by introducing a new variable

$$x_0(t) = \int_{t_0}^{t} f_0(x,u)dr \tag{10}$$

As a result, we get an extended set of equations of state

$$x_t = f_i(\boldsymbol{x}, \boldsymbol{u}), \quad i = 0, 1, ..., n \tag{11}$$

An additional vector, called the vector of feedback state, is also introduced

$$\boldsymbol{p} = [p_0, p_1, ..., p_n]^T \tag{12}$$

The vector is defined by a set of equations

$$\dot{p}_i = -\sum_{j=0}^{n} \frac{\partial f_j}{\partial x_i} p_j, \quad i = 0, 1, ..., n \tag{13}$$

and the scalar function

$$H(\mathbf{x},\mathbf{u},\mathbf{p}) = \sum_{j=0}^{n} \frac{\partial f_j}{\partial x_i} p_j, \quad i = 0, 1, ..., n \tag{14}$$

called hamiltonian.

With the use of Hamiltonian H we can present equations of state (11) and equations of feedback variables (13) in the form of canonical equations

$$\dot{x}_i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = \frac{\partial H}{\partial x_i}, \quad i = 0, 1, ..., n \tag{15}$$

The Pontriagin maximum principle defining the sufficient condition of control optimality, $U(t)$, says that for the admissible control $\mathbf{u} \in \Omega)$ to be optimal in the sense of a coefficient minimum (9) it is required for hamiltonian H, as a function u, to reach the maximum value.

The problem of the dynamic optimization can be considered as a problem of minimalization in the final point $t = t_k$ of the Pontragin function

$$P = \sum_{i=0}^{i=n} b_i x_i(t) \tag{16}$$

Coefficients b_i constitute final conditions for the equation (13), that is

$$p_i(t_k) = -b_i \tag{17}$$

In case of the final state which is set and limited by the following condition

$$R_j[x(t_k)] = 0, \quad j = 0, 1, ..., n \tag{18}$$

the Pontriagin function is presented in the form

$$P = \sum_{i=0}^{n} b_i x_i \sum_{j=0}^{1} \lambda_j R_j[x(t_k)] \tag{19}$$

where λ_j – Lagrange multipliers. The final conditions for the feedback vector now take the following form

$$p(t_k) = -\left[b_i + \sum_{j=0}^{1} \lambda_j \left. \frac{\partial R_j}{\partial x_i} \right|_{t=t_k} \right] \tag{20}$$

For linear systems the maximum principle constitutes at the same time the sufficient condition of control optimality.

Another method of determining the optimal control $u^n(t)$ is derived from the Bellman (23) optimality principle.

The optimality principle can be formulated in the following way:

If a trajectory is an optimum trajectory $x^0(t)$ *in the time interval* $[t_0, t_k]$, *then each part of it corresponding to the time interval* $[t_1, t_k]$ $(t_0 < t_1 < t_k)$ *is also an optimum trajectory for initial conditions satisfying the trajectory equation in the* t_1 *moment of time.*

From the optimality principle it follows that an optimum trajectory starting from a given point is fully defined by the initial state and does not depend on controls which caused the object to reach the state.

If $S(x,t)$ is the minimum value of the quality coefficient (9) on the final part of the optimal trajectory defined from a certain state $x(t)$ to the final state $x(t_k)$, that is

$$S(x,t) = \min_u \int_t^t f_0(x,u)dt \tag{21}$$

then the optimal control $u_0(t)$ $(t_0 < t < t_k)$ can be determined by solving the Bellman equation with the use of the dynamic programming method.

$$-\frac{\partial S}{\partial t} = \min_u \left[f_0(x,u) + \sum_{j=0}^{n} \frac{\partial S}{\partial x_i} f_i(x,u) \right] \tag{22}$$

The algorithm of the dynamic programming is very general and, in the essence, it does not require any assumptions related to the *f* function. It is suitable for defining a control series both in the closed and the open system. (cf. Fig. 4 and 5). In his study the author (Tymiński 2013: 104) presents an algorithm of dynamic programming applied for the optimization of the investment portfolio of shares on the capital market of three financial instruments (GTC, RPC and WWL stock):

$$\max F_{i,1+i}(x(t)) = \max[\, fz(tx) + f_{1+i}(x(t) - tx)] \tag{23}$$

where: $i = 1, 2, 3$; $x(t)$ – the sum of disposable resources in particular stages the optimization process, tx – resources (stock) is allocated to particular companies.

References

Achelis S.B. (1998), *Analiza techniczna od A do Z*, Oficyna Wydawnicza LT&P, Warszawa.
Athaus N., Falb P. (1969), *Sterowanie optymalne*, WNT, Warszawa.
Baborski A., Duda M., Forlicz S. (1983), *Elementy cybernetyki ekonomicznej*, PWE, Warszawa.
Bellman R. (1957), *Dynamic Programining*, Princeton University Press.
Bołtiański W.G. (1971), *Matematyczne metody sterowania optymalnego*, WNT, Warszawa.
Bukietyński W. (1974), *Elementy sterowania nieliniowego i dynamicznego*, WSE, Wrocław.
Carolan C. (1996), *Kalendarz spiralny*, WIG-Press, Warszawa.
Dworecki Z. (1989), *Zarządzanie strategiczne. Rodzaje metod portfelowych*, „Przegląd Organizacji" nr 6.
Gierałtowska U. (2004), *Wykorzystanie funkcji dyskryminacji do podejmowania optymalnych decyzji*, ed. T. Trzaskalik, Akademia Ekonomiczna im. Karola Adamieckiego w Katowicach, Katowice.
Jędrzejczyk Z., Kukuła K., Skrzypek J., Walkosz A. (2004), *Badania operacyjne w przykładach i zadaniach*, Wydawnictwo Naukowe PWN, Warszawa.
Kisperska-Moroń D., Krzyżaniak S. (2009), *Logistyka*, ILiM, Poznań.
Murphy I. (1999), *Analiza techniczna rynków finansowych*, Wydawnictwo WIG-Press, Warszawa.
Pluta W. (2000), *Budżetowanie kapitałów*, PWE, Warszawa.
Pońsko P. (2000), *Optymalizacja dynamiczna wzrostu gospodarczego.* Wyd. ELIPSA, Warszawa.
Rothschild I. (2000), *Księga bessy*, WIG-Press, Warszawa.
Rumatowski K., Królikowski A., Kasiński A. (1984), *Optymalizacja układów sterowania. Zadania*, WNT, Warszawa.
Tarczyński W. (2002), *Fundamentalny portfel papierów wartościowych*, PWE, Warszawa.
Tymiński J. (2011), *Podstawy procesów decyzyjnych na rynku kapitałowym*, WSGK, Kutno.
Tymiński J. (2013), *Ekonomiczne aspekty optymalizacji inwestycji długookresowych*, Wieś Jutra, Warszawa.

PROCESY STEROWANIA W ASPEKCIE LOGISTYCZNYCH DECYZJI EKONOMICZNYCH NA RYNKU KAPITAŁOWYM

Streszczenie. Artykuł przedstawia zarys procesów sterowania w aspekcie ekonomicznych decyzji logistycznych na rynku kapitałowym. W artykule przeprowadzona jest prezentacja matematyczna procesów sterowania obiektami gospodarczymi i kierunki sterowania optymalnego w układach otwartych i sprzężenia zwrotnego. Na przykładzie numerycznym z rynku kapitałowego, zilustrowany jest proces sterowania w ujęciu analizy technicznej. Autor w końcowych sekwencjach artykułu, przytacza pozycję bibliograficzną, w której jest zastosowany algorytm programowania dynamicznego do optymalizacji inwestycyjnego portfela długookresowego akcji na rynku kapitałowym.

Słowa kluczowe: sterowanie, optymalizacja, analiza techniczna, programowanie dynamiczne, logistyka

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