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Modelling Leverage Effect in a Financial Time Series

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Abstract: Economists and econometricians have been interested in modelling and forecasting financial instruments pricing since the very beginning of exchange trading. In more than one hundred years of history, a lot of different approaches have been developed in this field. The achievements gained in the last few years is the treatment of data from successive quotation as a time series indicating some specific properties. One property is the leverage effect.

Keywords: analysis of time series, modelling a leverage effect, the GARCH model

Introduction

Modern techniques of market risk management (the risk of changes in financial instruments prices) are based primarily on models coming from the theory of stochastic processes. By means of stochastic processes, changes in financial instruments prices are described (including the changes in interest rates) or a return on instruments. The choice of the model depends on what observable properties should it describe in a real time series. The aim of this paper is to present the basic techniques of the description of the leverage effect occurring in a series of stock returns and stock indices. The most popular models, which are often used afterwards to more complicated issues (risk measurement, a valuation of the option etc.), were chosen for analysis.

1. The leverage effect

The leverage effect means the phenomenon of a correlation of past returns with future volatility. This correlation is negative, which means that the variance of returns increases with a decrease in prices. The volatility estimator at the moment t with n time horizon, a standard deviation from a trial σ was adopted, where μ – sample mean, n – number of observations (22-day time horizon was adopted). Despite the fact that such a way of presenting the leverage effect raises many methodological doubts (21 previous observations depend on the volatility at the moment t) (Kothari, Warner 2009: 69). The measure of the leverage effect

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is the correlation of returns with successive squares of returns, which can be described in the following way:

$$L(k) = corr \ \{(x_{t+k})^2 , x_t\}.$$

$L(k)$ takes on non-negative values and falls to 0, which means that decreases in returns cause an increase in volatility. The leverage effect is asymmetrical, which means that the decrease in the value of returns leads to a much bigger increase in volatility than an analogical increase for an absolute value. Asymmetry, assuming that the total of information I_t flowing into the market at the t moment can be defined in the following way: *the return* r_t *indicates asymmetric volatility if:*

$$var(r_{t+1} \mid I_t, \check{n}_t, < 0) - \sigma_t^2 > var \ (r_{t+1} \mid I_t, \check{n}_t, > 0) - \sigma_t^2,$$

where var(.) means variance, σ_t – standard deviation. The above inequality means that unforeseen negative values of the return cause larger increases in the conditional volatility than positive values, which in particular may even lead to a decrease in the conditional volatility (Bekaert, Wu 2000: 1–42).

2. Theoretical approach of the leverage effect - leverage hypothesis

F. Black in 1976 was the first person who paid attention to the leverage effect. He noticed that volatility increases when prices decrease and tends to decline when prices rise. He proposed the following theoretical explanation of this phenomenon: the decline in a share price is identical with the decline of the market value of the equities of a company (which corresponds to the number of shares multiplied by the market value – the price). The market value of debt remains unchanged, but the ratio of debt/equity varies, causing the increase in the leverage effect. It means the increase in the company risk which is fully transferred to shareholders because of the limited claims of creditors. The equity of a company becomes more risky, which results in the increase in risk measured by share price volatility (Bollerslev et al. 1994: 122–128). On the contrary, when share price increases, the degree of the leverage effect decreases, resulting in the decrease in risk. It follows the following cause and effect relationship: the change in share price (a return) will change the future volatility (variance).

An undoubted advantage of the leverage hypothesis is that it perfectly explains the presence of variance asymmetry in the series of stock returns based on the prices of shares of indices and derivatives on shares, with the lack of the phenomenon in other financial times (for example, returns on commodities, currencies).

3. Criticism of the leverage hypothesis

The theory put forward by Black gained popularity due to its simplicity and logical clarity: it is much easier to understand the cause and effect relationship proposed in the leverage hypothesis' direction, which means that past fluctuations of returns affect the future volatility (risk) of the instrument (Kothari, Warner 2009: 52). The relationship of an opposite direction is less obvious. It is worth mentioning that the empirical verification of the leverage hypothesis is difficult to carry out: examination of the relationship between the effect and the level of the leverage hypothesis of companies requires knowledge of the market value of the debt (Gourieroux 1997: 17). Given the unavailability of such data for the size describing the debt of the company, the nominal value of debt is usually adopted. Despite the leverage hypothesis' assets, it is only a proposition of the theoretical conceptualization of the problem. Its main drawback is the fact that, it does not explain the variance asymmetry (Hentschel 1995: 71-104). The same changes in instrument price regarding absolute value should equally affect the level of financial leverage. However, the practice provides strong evidence that the phenomenon, to a much greater extent, takes place in cases of stock market declines. Price rises do not have to necessarily lead to the decline in volatility, on the contrary, a "reverse" leverage effect may be observed, i.e. the increase in volatility after a price rise. Sometimes, the concept of the down market effect is even postulated due to the strong presence of the phenomenon in the case of the decline in the market and its lack or statistical absence of significance during the bull market (Figlewski, Wang 2000). In addition, the financial leverage is a variable whose current level is important and not the change of its value since the last period. Therefore, a permanent change in the level of financial leverage should result in a permanent change of share volatility. However, it turns out that the leverage effect is not permanent and it expires within time (Figlewski, Wang 2000). If goodwill is the sum of debt and the number of shares multiplied by their market price, then the change of any of these three factors should cause changes in the financial leverage. From a theoretical point of view, the strength of the impact of market change in a share price, debt-increasing or issue of new shares should be the same. However, empirical data shows an unequal impact of changes in capital structure. Changes in debt value and the number of shares do not have a significant impact on the volatility of the company's shares. Equity value plays only a key role, which cannot be explained by Black's theory.

A key role is played only by the value on the basis of the theory which Black provided cannot be explained. The Leverage hypothesis does not explain also the presence of a stronger effect on the indices of shares (compared to their constituent companies) and on implied volatility. The assumption about the complete knowledge of investors on the current level of the ratio of debt/equity of selected companies may also raise doubts. The leverage hypothesis formulated by F. Black, even though it is very widespread has some serious weak points. It explains only partially future price volatility. Currently, volatility feedback effect has become a much more popular theory.

4. The volatility feedback effect

The volatility feedback effect has been mentioned many times as a possible theoretical explanation of the leverage effect. However, a full conceptualization was made by Y. Campbell and L. Hentschel, by modelling asymmetric dependence stock returns and volatility by means of QGARCH (Quadratic GARCH, E. Setana, 1991) (Campbell, Hentschel 1992).

The feedback effect is based on the following dependence: if volatility has its market price, anticipated increase in volatility will increase in the return required by investors. Suppose that the market is reached by a large amount of bad information on future dividends. This results in a decline in stock prices. Because variability shows a long-term dependence on the factors affecting it (or a long memory), the information will affect the growth of both current and future volatility (Jajuga 2000: 93). Increased and expected volatility of the instrument means an increase in its risk, for which investors expect a higher return. In turn, these are obtainable only when the current price of the instrument will fall. In this way, the volatility feedback effect reinforces the initial reaction of prices. Now, let us examine the opposite situation: good news reaches the market. It influences an increase in the price of the instrument but its appearance also results in an increase in future volatility, which must be covered by a higher rate of return. The feedback in this case undermines the original positive impact on the price information. In this case, the feedback undermines the original positive impact of information on a price (Kothari, Warner 2009: 69).

The feedback effect implies a cause and effect relationship reversed in relation to the leverage hypothesis: future volatility influences the current price of the instrument. The volatility feedback effect is also known as the time-varying risk premium theory. It is worth mentioning that the concept does not refer to any model of capital market equilibrium, unlike some attempts of model specifications within it (Bekaert, Wu 2000: 1–42). The only thing it requires is a positive dependence between conditional volatility and an expected return. However, this assumption raises a lot of controversy.

5. The properties of a financial time series of returns

The following part of the study concerns only typical solutions for the analysis of a series of returns from share prices and share indices. Only in these series, characteristics described in this paper can be observed. Empirical research on the series of returns from share prices and share indices lead to rejecting the hypothesis that the prices of these instruments vary according to the geometric Brownian motion of constant parameters. In many works (see Box, Jenkins (1986), Bollerslev (1986), and Tsay (2002)), the results of empirical studies were presented for different instruments which contradict this assumption.

These studies showed a presence in the series of returns:

– the volatility clustering effect, which is an instability of variance (Gourieroux 1997: 24) of returns over time,

- the leptokurtosis effect and thicker tails of distribution returns from a normal distribution,
- the skewness effect of distributions of returns,
- the autocorrelation effect of returns,
- the leverage effect – the asymmetric impact of positive and negative information on the level of future variance,
- the effect of long memories in the series of volatility.

6. The GARCH model, as a basis for modelling the conditional variance of a financial series

Due to the presence of specific properties in a financial time series (the leverage effect, the accumulation of variability, a long memory, thick tails, leptokurtosis etc.), it was necessary to develop a new approach to model financial data. GARCH models belong to this approach. In 1982, R. Engle proposed the model ARCH (Autoregressive Conditional Heteroskedastic Model), he was the first person who made a dependent current variance of the process on its value in previous periods. Thanks to this, it became possible to describe collecting variations (Bollerslev et al. 1994: 37). In addition, ARCH enables modelling leptokurtosis and thick tails of unconditional distributions of returns. Model GARCH (p, q) (Generalized ARCH Model) was proposed in 1986 by T. Bollerslev, as a generalization of the ARCH model (q). So far the considered models are symmetric and do not include the leverage effect – positive and negative information shocks have the same impact on the conditional variance (Bollerslev et al. 1994: 37). However, every model can be generalized by introducing asymmetry. A news impact curve is a simple tool enabling the achievement of this goal (Jajuga 2000: 62). This approach has been proposed by Engle and Ng (1993) and consists in replacing the rest of the model by their transformation. Thus, a news impact curve is a function assigning appropriate amounts to the conditional variance to the values of a random component:

$$\sigma_t^2 = f(\varepsilon_{t-1}).$$

The asymmetry of the news impact curve can be in two ways:

- by shifting the symmetric function in a such way that its minimum is not at point $\varepsilon_{t-1} = 0$,
- through the rotation of a curve, that is staying the minimum of function at point $\varepsilon_{t-1} = 0$, but the introduction of asymmetry in the slope of its shoulders.

These approaches differ from each other and should not be regarded as substitutes. Both methods can be used together, at that moment, and they can enhance their influence or endure. By means of the Box-Cox transformation, the classical GARCH model can be transformed in a general asymmetric form enabling the modelling leverage effect (Hentschel 1995: 71–104).

The classic AR-GARCH model

Before the presentation of the most popular solutions regarding the modelling of the leverage effect, it is essential to introduce the basic information on the general concept of the description of the series returns and the classic AR-GARCH model (Hentschel 1995: 71–104). In the following part of the work, the model in a discrete time describing the time series of simple returns is presented by an equation.

The construction of a good model of a series of returns should include:
- the choice of the form and function of density of standardized model residuals,
- modelling of the conditional expected value of the process,
- modelling of the conditional variance of the process.

All three issues need to be considered together, as they mutually influence each other and they together determine the properties of the final model.

Standard model residuals

In basic versions of the models proposed by Engle and Bollersleva (cf. Bollerslev, Engle, Nelson (1994) standard model residuals (zt) and empirical studies have shown that the actual model residuals have conditional distributions of thicker tails than a normal distribution. Therefore, a number of innovations were proposed in this field. The following schedules are the most often used[1]: The General Error Distribution, GED), slanting and symmetrical *t*-Student schedule has fewer extreme values. To maintain conceptual consistency in terms of modelling the conditional expected value and conditional variance with the classic AR-GARCH model, the possibility of scaling the used schedules must be possible so as to achieve the zero mean and a unit variance (Bollerslev et al. 1994: 122–128).

Conditional expected value

The introduction of a conditional expected value to the model enables to take into account the autocorrelation effect in the series of returns in a simple and elegant way. In the majority of the series of stock returns and above all index returns, a significant, positive value of the autocorrelation of the first order can be observed. Significant autocorrelations of higher orders than the first one are rare and most often are negative (see Jajuga 2000; Tsay 2002). The known processes from the class of linear processes of auto regression and moving average (ARMA) (see Box, Jenkins 1986; Milo 1990) are used to describe observable autocorrelations of the series of returns. It should be noted that the pure models $AR(m)$ are the most

[1] Except the normal schedule which still dominates.

often applied because the observable variations (r_{t-k}) are used in them, and not observable variables (ε_{t-k}) like in the case of moving average and mixed models. Typically, the *AR*(1) process is used and its conditional expected value is given by the formula:

$$\mu_t = \mathrm{E}\left[r_t \mid I_{t-1}\right] = \mu_0 + \varphi_1 r_{t-1}.$$

This model is used also in an empirical part to describe the series of returns on the WIG index. In the case of the Polish market, applying models of higher orders is inadvisable (insignificant values of coefficients for the model of a higher order than 1).

7. Analysis of the leverage effect in the series of stock returns listed in the Warsaw Stock Exchange

The series of share prices of the 50 largest and most liquid companies listed on the Warsaw Stock Exchange have been analysed, and for these companies, it can be expected that the market valuation is the most effective[2]. To ensure the sufficient length of a time series, only those companies listed at since January 1, 2004 were included. On the basis of the prices of chosen shares at the end of each trading day, simple rates of return were calculated. A 10-year time series for the period from 1 January 2004 to 31 December 2013 were chosen. Therefore each series of share prices contains about 2,500 observations. For the data analysis, 6 selected models of the GARCH class were chosen: a simple GARCH model and three asymmetric models as a result of the transformation: EGARCH, GJR GARCH, AP (G) ARCH and two asymmetric models with a long memory: FIEGARCH and FIAPGARCH.

8. Model estimation

This paper focuses on the leverage effect. However, time series properties known from literature cannot be omitted. For this reason and in order not to complicate the analysis, the lowest rows of models have been chosen (i.e. AR (1) and GARCH (1,1)) and a slanting Student-t schedule was adopted arbitrarily. These assumptions simplify the estimation, without affecting the further considerations. They are a compromise between the wish to select the best models and a clearly specified objective of the research. Eventually the following models were selected:

AR(1)-GARCH(1,1) – slanting t-Student,
AR(1)-EGARCH(1,1) – slanting t-Student,
AR(1)-GJR GARCH(1,1) – slanting t-Student,

[2] The choice of companies was made on the basis of the following index: WIG20, mWIG40 and sWIG80 so as to choose only these companies which are characterized by a relatively large and regular turnover. Financial institutions were excluded, banks, brokerage houses, investment funds and the stock exchange), the level of the financial leverage may be considerably different from the market average.

AR(1)-AP(G)ARCH(1,1) – slanting t-Student,
AR(1)-FIEGARC(1,d,1) – slanting t-Student,
AR(1)-FIAPGARCH(1,d,1) – slanting t-Student,
Altogether 300 models were estimated by the method of the highest credibility.

9. Price and comparison of models

The higher the value of the log-likelihood function (LLF), the better the model. However, almost always, a more complex model has the greater value LLF than an easier model. Therefore, during the evaluation and comparison of the models, the number of parameters should be taken into account (Gourieroux 1997: 17). With the increase in the number of parameters, the complexity of the model increases, which is a major drawback in its practical use. Estimated models were compared by the means of the following information criteria:

1. Akaike information criteria

$$AIC = \frac{-2LLF + 2\ (\text{number of model parameters})}{\text{number of observations}}.$$

2. Schwartz information criteria

$$SIC = \frac{-2LLF + 2\ln\ (\text{number of model parameters})}{\text{number of observations}}.$$

3. Hannan-Quinn information criteria

$$HQIC = \frac{-2LLF + 2\ (\text{number of model parameters})\ \text{in}\ (\text{number of observations})}{\text{number of observations}}.$$

The lower the value of the information criterion, the better the model. For models nested in pairs, it could be possible to make a comparison by using the LRT test:

$$LRT = 2(LLF_1 - LLF_0),$$

where:

LLF_1 – log-maximum likelihood value for the model with a bigger number of the parameter,

LLF_0 – log-maximum likelihood value for the model with a smaller number of a parameter. The H null hypothesis of the LRT test assumes that a simpler model is better with a smaller number of parameters. LRT has a $\chi 2$ distribution with the number of freedom degrees equal to the number of parameters which the compared models differ by.

10. Research results

The Model FIAPGARCH turned out to be by far the best in terms of value of information criteria.

Table 1

Number of responses of the given models by the information criteria's AIC, SIC and HQIC

Model	AIC	SIC	HQIC	Total
GARCH	1	0	2	3
EGARCH	2	1	1	4
GJR-GARCH	1	1	1	3
AP(G)ARCH	3	3	4	10
FIEGARCH	7	7	7	21
FIAPGARCH	36	35	32	103

Source: own calculation.

Only in three cases, the information criteria indicated different models.[3] LRT tests in 47 cases confirmed responses to the information criteria.[4]

Conclusions

The leverage effect constitutes an important part of the modelling of the conditional variance of a series of returns from listed Polish companies. For a sample of 50 companies, only 2 do not show the presence of the phenomenon. The introduction of asymmetry to the model sufficiently improves its quality, that is why; the lever modelling stage should not be overlooked. Knowledge concerning the presence of the leverage effect should be developed by a deeper quality analysis of known models and by studying the shape of the relation between returns and volatility for the specification of an optimal model. FIAPGARCH seems to be the best model for the series of returns. This is not surprising – the model (from the all mentioned) has the most flexible (in terms of the possibility to match data) news impact curve. Also, it is worth drawing attention to the prevalence of "a long memory" effect. Using the models of the GARCH class to the Polish market, the one which allows modelling the leverage and a long memory effect at the same time should be chosen.

[3] The models indicated twice were adopted for two companies.

[4] In the case of one of the companies, LRT indicates the model AP(G)ARCH. Unfortunately, it is not possible to compare it with the model FIEGARCH. Undoubtedly, for this company, the leverage effect occurs, however a closer specification of the model is risky. Therefore, one of the models was adopted without indicating which one.

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MODELOWANIE EFEKTU DŹWIGNI W FINANSOWYCH SZEREGACH CZASOWYCH

Streszczenie: Zainteresowanie ekonomistów i ekonometryków modelowaniem i prognozowaniem cen instrumentów finansowych towarzyszy od samego początku obrotowi giełdowemu. W ciągu ponad stuletniej historii wypracowano w tym zakresie wiele różnych podejść. Dorobek ostatnich kilkudziesięciu lat to traktowanie danych z kolejnych notowań jako szeregi czasowe, wykazujące pewne specyficzne właściwości. Jedną z takich własności jest efekt dźwigni.

Słowa kluczowe: analiza szeregów czasowych, modelowanie efektu dźwigni, model GARCH

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